Transportation Problem (TP)

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REVIEW

Studi Kasus 2

Sebuah perusahaan logistik memiliki empat pabrik yang menghasilkan produk untuk didistribusikan ke 3 kota. Perusahaan ingin menentukan jumlah produk yang dikirim dari setiap pabrik ke setiap kota untuk meminimalkan biaya bahan bakar, dengan mempertimbangkan kapasitas produksi pabrik dan permintaan kota.

- Kapasitas pabrik 1, pabrik 2, pabrik 3, dan pabrik 4 adalah 140, 260, 360, dan 220 ton.
- Permintaan kota 1, kota 2, dan kota 3 adalah 200, 320, dan 250 ton.
- Masing-masing pabrik dapat mengirimkan produk ke masing-masing kota (tidak ada rute yang tidak tersedia).

Studi Kasus 2

Tentukan *Decision variable*:, *objective function*, dan *constraints* dari studi kasus tersebut.

Adapun biaya pengiriman per unit adalah:

- Pabrik 1 ke kota 1: 48
- Pabrik 1 ke kota 2: 60
- Pabrik 1 ke kota 3: 56
- Pabrik 2 ke kota 1: 45
- Pabrik 2 ke kota 2: 55
- Pabrik 2 ke kota 3: 53
- Pabrik 3 ke kota 1: 50
- Pabrik 3 ke kota 2: 65
- Pabrik 3 ke kota 3: 60
- Pabrik 4 ke kota 1: 52
- Pabrik 4 ke kota 2: 64
- Pabrik 4 ke kota 3: 55

Studi Kasus 2

- *Decision variable*:
  `xij` (jumlah barang yang dikirimkan dari pabrik ke-i menuju kota ke-j).
  `i = 1, 2, 3, 4`
  `j = 1, 2, 3`

- Objective function:
  `min Z = 48*x11 + 60*x12 + 56*x13 + 45*x21 + 55*x22 + 53*x23 + 50*x31 + 65*x32 + 60*x33 + 52*x41 + 64*x42 + 55*x43`

Studi Kasus 2

Constraints:

- Kapasitas pabrik

$$x11 + x12 + x13 \leq 140$$
$$x21 + x22 + x23 \leq 260$$
$$x31 + x32 + x33 \leq 360$$
$$x41 + x42 + x43 \leq 220$$

- Permintaan kota

$$x11 + x21 + x31 + x41 \leq 200$$
$$x12 + x22 + x32 + x42 \leq 320$$
$$x13 + x23 + x33 + x43 \leq 250$$

Studi Kasus 2

	Kota 1 (D1)	Kota 2 (D2)	Kota 3 (D3)	Supply
Pabrik 1 (S1)	48	60	56	s1 = 140
Pabrik 2 (S2)	45	55	53	s2 = 260
Pabrik 3 (S3)	50	65	60	s3 = 360
Pabrik 4 (S4)	52	64	55	s4 = 220
Demand	d1 = 200	d2 = 320	d3 = 250	

Penyelesaian TP

Metode Penentuan Solusi Awal (*Initial Basic Feasible Solution*)

- North West Corner Method (NWCM)
- Least Cost Method (LCM)
- Vogel's Approximation Method (VAM)

Metode Penentuan Solusi Optimal

- *Stepping Stone*
- *Modified Distribution*

Metode Penentuan Solusi Awal (*Initial Basic Feasible Solution*)

North West Corner Method (NWCM)

Least Cost Method (LCM)

Vogel’s Approximation Method (VAM)

Metode Penentuan Solusi Optimal

- *Stepping Stone*
- *Modified Distribution*

Agar suatu solusi awal / *Initial Basic Feasible Solution* (IBFS) bisa dioptimalkan, maka IBFS perlu memenuhi persyaratan sbb:

$$\text{Jumlah cell yang memiliki alokasi (}\textit{allocated cell}\text{)} = m+n-1$$

Dimana m = jumlah node asal / *source* node dan n = jumlah node tujuan / *destination* node

North West Corner Method

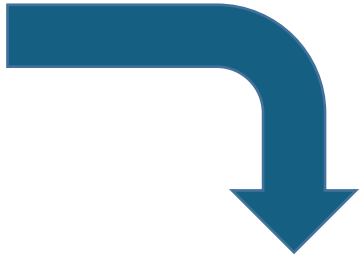
North West Corner Method

- NWCM adalah salah satu teknik dalam **penyelesaian masalah transportasi** di riset operasional. Metode ini digunakan untuk menentukan **solusi awal yang layak** dalam mendistribusikan barang dari beberapa sumber ke beberapa tujuan, tanpa langsung mempertimbangkan biaya pengiriman.
- Metode ini dinamai berdasarkan cara kerjanya:
  ➡️ Alokasi dimulai dari **sel paling kiri atas** (sudut barat laut) dari matriks transportasi, lalu bergerak ke kanan atau ke bawah sesuai kondisi supply dan demand.

Algorithm

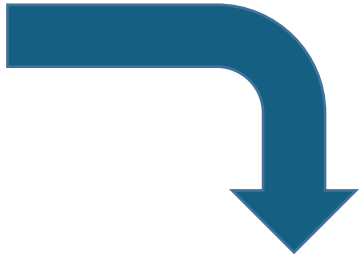
North-West Corner Method (NWCM) Steps (Rule)	
Step-1:	Select the upper left corner cell of the transportation matrix and allocate min(s1, d1).
Step-2:	a. Subtract this value from supply and demand of respective row and column. b. If the supply is 0, then cross (strike) that row and move down to the next cell. c. If the demand is 0, then cross (strike) that column and move right to the next cell. d. If supply and demand both are 0, then cross (strike) both row & column and move diagonally to the next cell.
Step-3:	Repeat this steps until all supply and demand values are 0.

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



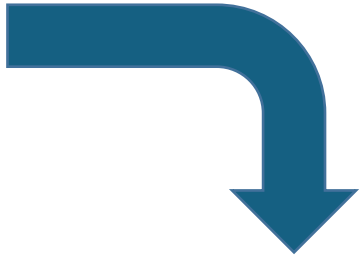
	D_1	D_2	D_3	D_4	Supply
S_1	19 (5)	30	50	10	2
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	8	7	14	

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10	2
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	8	7	14	



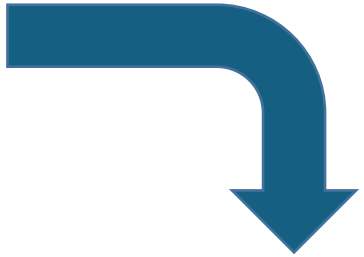
	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19 **(5)**~~	~~30 **(2)**~~	~~50~~	~~10~~	0
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	6	7	14	

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	0	6	7	14	



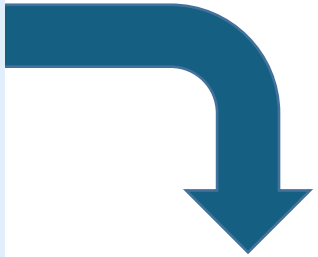
	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
S_2	70	30(6)	40	60	3
S_3	40	8	70	20	18
Demand	0	0	7	14	

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19 (5)~~	~~30 (2)~~	~~50~~	~~10~~	0
S_2	70	30 (6)	40	60	3
S_3	40	8	70	20	18
Demand	0	0	7	14	



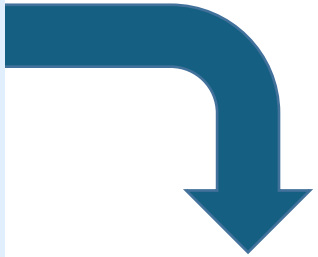
	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19 (5)~~	~~30 (2)~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30 (6)~~	~~40 (3)~~	~~60~~	0
S_3	40	8	70	20	18
Demand	0	0	4	14	

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30(6)~~	~~40(3)~~	~~60~~	0
S_3	40	8	70	20	18
Demand	0	0	4	14	



	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19(5)~~	~~30(2)~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30(6)~~	~~40(3)~~	~~60~~	0
S_3	40	8	70(4)	20	14
Demand	0	0	0	14	

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19 **(5)**~~	~~30 **(2)**~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30 **(6)**~~	~~40 **(3)**~~	~~60~~	0
S_3	40	8	70 **(4)**	20	14
Demand	0	0	0	14	



	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19 **(5)**~~	~~30 **(2)**~~	~~50~~	~~10~~	0
~~S_2~~	~~70~~	~~30 **(6)**~~	~~40 **(3)**~~	~~60~~	0
~~S_3~~	~~40~~	~~8~~	~~70 **(4)**~~	~~20 **(14)**~~	0
Demand	0	0	0	0	

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30 **(2)**	50	10	7
S_2	70	30 **(6)**	40 **(3)**	60	9
S_3	40	8	70 **(4)**	20 **(14)**	18
Demand	5	8	7	14	

The minimum total transportation cost =19×5+30×2+30×6+40×3+70×4+20×14=1015

Least Cost Method

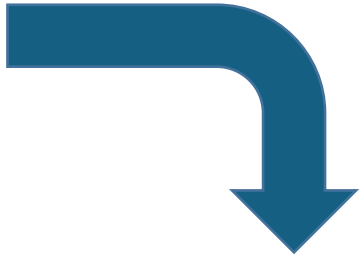
Least Cost Method (LCM)

- Least Cost Method (LCM) adalah salah satu teknik dalam **masalah transportasi** yang digunakan untuk menentukan **solusi awal yang layak** dengan cara **meminimalkan total biaya pengiriman** sejak awal alokasi.
- Metode ini bekerja dengan prinsip:
  ➡️ **Alokasikan barang ke sel dengan biaya transportasi paling rendah terlebih dahulu**, lalu lanjutkan ke sel dengan biaya terendah berikutnya, sampai semua supply dan demand terpenuhi.

Algorithm

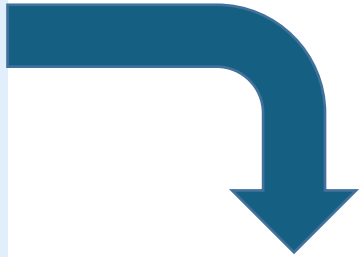
Least Cost Method (LCM) Steps (Rule)	
Step-1:	Select the cell having minimum unit cost c_{ij} and allocate as much as possible, i.e. $\min\left(s_i, d_j\right)$.
Step-2:	a. Subtract this $\min$ value from supply s_i and demand d_j. b. If the supply s_i is 0, then cross (strike) that row and If the demand d_j is 0 then cross (strike) that column. c. If min unit cost cell is not unique, then select the cell where maximum allocation can be possible
Step-3:	Repeat this steps for all uncrossed (unstriked) rows and columns until all supply and demand values are 0.

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



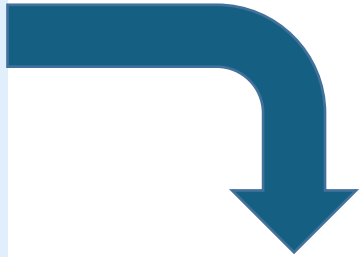
	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8(B)	70	20	10
Demand	5	0	7	14	

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8(B)	70	20	10
Demand	5	0	7	14	



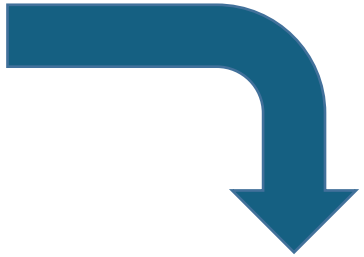
	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40	60	9
S_3	40	8(B)	70	20	10
Demand	5	0	7	7	

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40	60	9
S_3	40	8(B)	70	20	10
Demand	5	0	7	7	



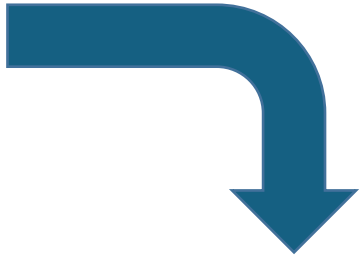
	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10(7)~~	0
S_2	70	30	40	60	9
S_3	40	8(B)	70	20(7)	3
Demand	5	0	7	0	

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10~~ (7)	0
S_2	70	30	40	60	9
S_3	40	8 (8)	70	20 (7)	3
Demand	5	0	7	0	



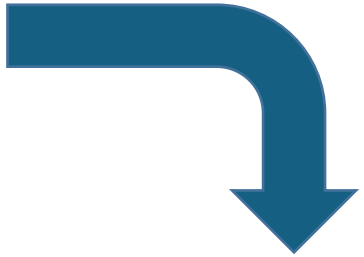
	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10~~ (7)	0
S_2	70	30	40 (7)	60	2
S_3	40	8 (8)	70	20 (7)	3
Demand	5	0	0	0	

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10~~(7)	0
S_2	70	30	40(7)	60	2
S_3	40	8(8)	70	20(7)	3
Demand	5	0	0	0	



	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10~~(7)	0
S_2	70	30	40(7)	60	2
~~S_3~~	~~40~~(3)	~~8~~(8)	~~70~~	~~20~~(7)	0
Demand	2	0	0	0	

	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10~~**(7)**	0
S_2	70	30	40**(7)**	60	2
~~S_3~~	~~40~~**(3)**	~~8~~**(3)**	~~70~~	~~20~~**(7)**	0
Demand	2	0	0	0	



	D_1	D_2	D_3	D_4	Supply
~~S_1~~	~~19~~	~~30~~	~~50~~	~~10~~**(7)**	0
~~S_2~~	~~70~~**(2)**	~~30~~	~~40~~**(7)**	~~60~~	0
~~S_3~~	~~40~~**(3)**	~~8~~**(3)**	~~70~~	~~20~~**(7)**	0
Demand	0	0	0	0	

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10 **(7)**	7
S_2	70 **(2)**	30	40 **(7)**	60	9
S_3	40 **(3)**	8 **(8)**	70	20 **(7)**	18
Demand	5	8	7	14	

The minimum total transportation cost =10×7+70×2+40×7+40×3+8×8+20×7=814

Vogel's Approximation Method

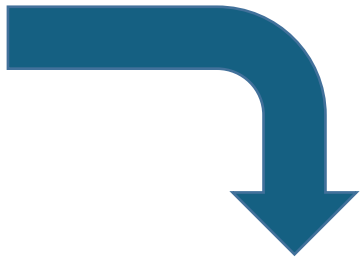
Vogel’s Approximation Method (VAM)

- Vogel’s Approximation Method (VAM) adalah teknik dalam **penyelesaian masalah transportasi** yang digunakan untuk mendapatkan **solusi awal yang lebih efisien**. Metode ini mempertimbangkan **biaya transportasi dan potensi kerugian** jika tidak memilih jalur terbaik.
- VAM bekerja dengan menghitung **“penalty” atau selisih biaya** antara dua pilihan terbaik dalam setiap baris dan kolom. Penalty ini menunjukkan **potensi kerugian** jika tidak memilih opsi termurah. Dengan cara ini, VAM membantu memilih jalur yang tidak hanya murah, tapi juga strategis.

Algorithm

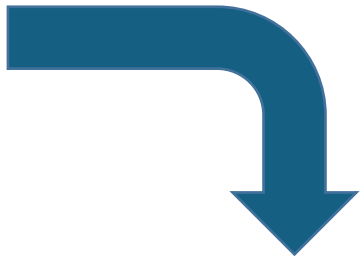
Vogel's Approximation Method (VAM) or penalty method This method is preferred over the NWCM and LCM, because the initial basic feasible solution obtained by this method is either optimal solution or very nearer to the optimal solution.	
Vogel's Approximation Method (VAM) Steps (Rule)	
Step-1:	Find the cells having smallest and next to smallest cost in each row and write the difference (called penalty) along the side of the table in row penalty.
Step-2:	Find the cells having smallest and next to smallest cost in each column and write the difference (called penalty) along the side of the table in each column penalty.
Step-3:	Select the row or column with the maximum penalty and find cell that has least cost in selected row or column. Allocate as much as possible in this cell. If there is a tie in the values of penalties then select the cell where maximum allocation can be possible
Step-4:	Adjust the supply & demand and cross out (strike out) the satisfied row or column.
Step-5:	Repeat this steps until all supply and demand values are 0.

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



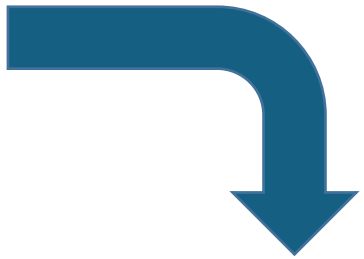
	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19	30	50	10	7	9 = 19 - 10
S_2	70	30	40	60	9	10 = 40 - 30
S_3	40	8	70	20	18	12 = 20 - 8
Demand	5	8	7	14		
Column Penalty	21 = 40 - 19	22 = 30 - 8	10 = 50 - 40	10 = 20 - 10		

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



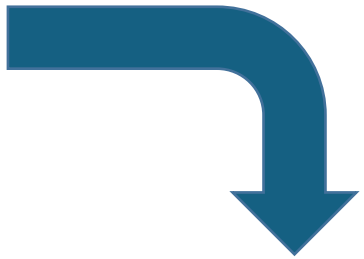
	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19	30	50	10	7	9 = 19 - 10
S_2	70	30	40	60	9	10 = 40 - 30
S_3	40	8	70	20	18	12 = 20 - 8
Demand	5	8	7	14		
Column Penalty	21 = 40 - 19	22 = 30 - 8	10 = 50 - 40	10 = 20 - 10		

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



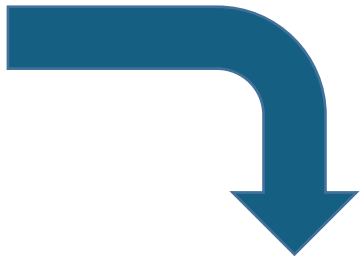
	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19	30	50	10	7	9 = 19 - 10
S_2	70	30	40	60	9	20 = 60 - 40
S_3	40	8(B)	70	20	10	20 = 40 - 20
Demand	5	0	7	14		
Column Penalty	21 = 40 - 19	--	10 = 50 - 40	10 = 20 - 10		

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



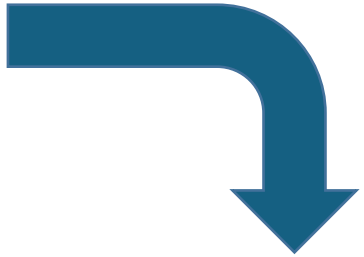
	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19 (5)	30	50	10	2	40 = 50 - 10
S_2	70	30	40	60	9	20 = 60 - 40
S_3	40	8 (3)	70	20	10	50 = 70 - 20
Demand	0	0	7	14		
Column Penalty	--	--	10 = 50 - 40	10 = 20 - 10		

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



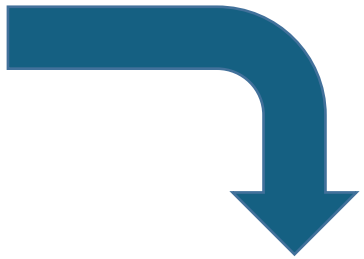
	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19(5)	30	50	10	2	40 = 50 - 10
S_2	70	30	40	60	9	20 = 60 - 40
~~S_3~~	~~40~~	~~8(8)~~	~~70~~	~~20(10)~~	0	--
Demand	0	0	7	4		
Column Penalty	--	--	10 = 50 - 40	50 = 60 - 10		

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



	D_1	D_2	D_3	D_4	Supply	Row Penalty
~~S_1~~	~~19(5)~~	~~30~~	~~50~~	~~10(2)~~	0	--
S_2	70	30	40	60	9	20 = 60 - 40
~~S_3~~	~~40~~	~~8(3)~~	~~70~~	~~20(10)~~	0	--
Demand	0	0	7	2		
Column Penalty	--	--	40	60		

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	



	D_1	D_2	D_3	D_4	Supply	Row Penalty
~~S_1~~	~~19~~ **(5)**	~~30~~	~~50~~	~~10~~ **(2)**	0	--
S_2	70	30	40	60 **(2)**	7	40
~~S_3~~	~~40~~	~~8~~ **(3)**	~~70~~	~~20~~ **(10)**	0	--
Demand	0	0	7	0		
Column Penalty	--	--	40	--		

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19**(5)**	30	50	10**(2)**	7	9 | 9 | 40 | 40 | -- | -- |
S_2	70	30	40**(7)**	60**(2)**	9	10 | 20 | 20 | 20 | 20 | 40 |
S_3	40	8**(8)**	70	20**(10)**	18	12 | 20 | 50 | -- | -- | -- |
Demand	5	8	7	14		
Column Penalty	21 21 -- -- -- --	22 -- -- -- -- --	10 10 10 10 40 40	10 10 10 50 60 --		

The minimum total transportation cost =19×5+10×2+40×7+60×2+8×8+20×10=779

Latihan

Dari Studi Kasus 2 tentukan metode penentuan solusi awal (*Initial Basic Feasible Solution*) dengan 3 metode tersebut

	Kota 1 (D1)	Kota 2 (D2)	Kota 3 (D3)	Supply
Pabrik 1 (S1)	48	60	56	s1 = 140
Pabrik 2 (S2)	45	55	53	s2 = 260
Pabrik 3 (S3)	50	65	60	s3 = 360
Pabrik 4 (S4)	52	64	55	s4 = 220
Demand	d1 = 200	d2 = 320	d3 = 250	