

Transportation Problem (Stepping Stone & MODI)

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Penyelesaian TP

Metode Penentuan Solusi Awal (*Initial Basic Feasible Solution*)

- North West Corner Method (NWCM)
- Least Cost Method (LCM)
- Vogel's Approximation Method (VAM)

Metode Penentuan Solusi Optimal

- *Stepping Stone*
- *Modified Distribution*

Stepping Stone

Stepping Stone

- ✓Metode ini digunakan untuk mengevaluasi apakah solusi awal sudah optimal atau belum.
- ✓Metode ini mengevaluasi **setiap sel kosong** (tidak dialokasikan) dalam tabel transportasi untuk melihat apakah memindahkan sebagian alokasi ke sel tersebut akan **mengurangi total biaya**.

Stepping Stone Method Steps (Rule)	
Step-1:	Find an initial basic feasible solution using any one of the three methods NWCM, LCM or VAM.
Step-2:	1. Draw a closed path (or loop) from an unoccupied cell. The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell. 2. Add the transportation costs of each cell traced in the closed path. This is called net cost change. 3. Repeat this for all other unoccupied cells.
Step-3:	1. If all the net cost change are ≥ 0, an optimal solution has been reached. Now stop this procedure. 2. If not then select the unoccupied cell having the highest negative net cost change and draw a closed path.
Step-4:	1. Select minimum allocated value among all negative position (-) on closed path 2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell). 3. Add this value to the other occupied cells marked with (+) sign. 4. Subtract this value to the other occupied cells marked with (-) sign.
Step-5:	Repeat Step-2 to step-4 until optimal solution is obtained. This procedure stops when all net cost change ≥ 0 for unoccupied cells.

Langkah Stepping Stone

1. Identifikasi sel kosong (tidak ada alokasi) dalam tabel transportasi.
2. Buat lintasan tertutup (closed path) dari sel kosong tersebut, dengan langkah zig-zag (horizontal dan vertikal) melalui sel yang sudah dialokasikan.
3. Tambahkan dan kurangi biaya secara bergantian di sepanjang lintasan tersebut.
4. Hitung perubahan biaya total jika satu unit dialokasikan ke sel kosong tersebut.
5. Evaluasi semua sel kosong dan pilih yang memberikan pengurangan biaya terbesar.
6. Lakukan re-alokasi sesuai lintasan tersebut jika ada pengurangan biaya.

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

1. Find initial solution (VAM / NWCR / LCM)

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19(5)	30	50	10(2)	7	9 | 9 | 40 | 40 | -- | -- |
S_2	70	30	40(7)	60(2)	9	10 | 20 | 20 | 20 | 20 | 40 |
S_3	40	8(8)	70	20(10)	18	12 | 20 | 50 | -- | -- | -- |
Demand	5	8	7	14		
Column Penalty	21 21 -- -- -- --	22 -- -- -- -- --	10 10 10 10 40 40	10 10 10 50 60 --		

total transportation cost
=19×5+10×2+40×7+60×2+
8×8+20×10
=779

Iteration 1

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30 ★	50 ★	10 **(2)**	7
S_2	70 ★	30 ★	40 **(7)**	60 **(2)**	9
S_3	40 ★	8 **(8)**	70 ★	20 **(10)**	18
Demand	5	8	7	14	

2. Create closed loop for unoccupied cells

Unoccupied cell	Closed path	Net cost change
S_1D_2	$S_1D_2 \rightarrow S_1D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 10 + 20 - 8 = 32
S_1D_3	$S_1D_3 \rightarrow S_1D_4 \rightarrow S_2D_4 \rightarrow S_2D_3$	50 - 10 + 60 - 40 = 60
S_2D_1	$S_2D_1 \rightarrow S_2D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	70 - 60 + 10 - 19 = 1
S_2D_2	$S_2D_2 \rightarrow S_2D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 60 + 20 - 8 = -18
S_3D_1	$S_3D_1 \rightarrow S_3D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	40 - 20 + 10 - 19 = 11
S_3D_3	$S_3D_3 \rightarrow S_3D_4 \rightarrow S_2D_4 \rightarrow S_2D_3$	70 - 20 + 60 - 40 = 70

Iteration 1

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30	40 **(7)**	60 **(2)**	9
S_3	40	8 **(8)**	70	20 **(10)**	18
Demand	5	8	7	14	

2. Create closed loop for unoccupied cells

Unoccupied cell	Closed path	Net cost change
S_1D_2	$S_1D_2 \rightarrow S_1D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 10 + 20 - 8 = 32
S_1D_3	$S_1D_3 \rightarrow S_1D_4 \rightarrow S_2D_4 \rightarrow S_2D_3$	50 - 10 + 60 - 40 = 60
S_2D_1	$S_2D_1 \rightarrow S_2D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	70 - 60 + 10 - 19 = 1
S_2D_2	$S_2D_2 \rightarrow S_2D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 60 + 20 - 8 = -18
S_3D_1	$S_3D_1 \rightarrow S_3D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	40 - 20 + 10 - 19 = 11
S_3D_3	$S_3D_3 \rightarrow S_3D_4 \rightarrow S_2D_4 \rightarrow S_2D_3$	70 - 20 + 60 - 40 = 70

Give positive and negative sign at each corner alternately

Iteration 1

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30	40 **(7)**	60 **(2)**	9
S_3	40	8 **(8)**	70	20 **(10)**	18
Demand	5	8	7	14	

2. Create closed loop for unoccupied cells

Unoccupied cell	Closed path	Net cost change
S_1D_2	$S_1D_2 \rightarrow S_1D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 10 + 20 - 8 = 32
S_1D_3	$S_1D_3 \rightarrow S_1D_4 \rightarrow S_2D_4 \rightarrow S_2D_3$	50 - 10 + 60 - 40 = 60
S_2D_1	$S_2D_1 \rightarrow S_2D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	70 - 60 + 10 - 19 = 1
S_2D_2	$S_2D_2 \rightarrow S_2D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 60 + 20 - 8 = -18
S_3D_1	$S_3D_1 \rightarrow S_3D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	40 - 20 + 10 - 19 = 11
S_3D_3	$S_3D_3 \rightarrow S_3D_4 \rightarrow S_2D_4 \rightarrow S_2D_3$	70 - 20 + 60 - 40 = 70

Select cell with the highest negative net cost change

3. Draw closed path from S2D2

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30 [32]	50 [60]	10 **(2)**	7
S_2	70 [1]	30 [-18] **(+)**	40 **(7)**	60 **(2)** **(-)**	9
S_3	40 [11]	8 **(8)** **(-)**	70 [70]	20 **(10)** **(+)**	18
Demand	5	8	7	14	

Select minimum allocated value among all negative position (-) on closed path = 2

4. Substract 2 from all (-) and Add it to all (+)

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30 **(2)**	40 **(7)**	60	9
S_3	40	8 **(6)**	70	20 **(12)**	18
Demand	5	8	7	14	

Repeat the step 2 to 4, until an optimal solution is obtained

Iteration 2

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30 **(2)**	40 **(7)**	60	9
S_3	40	8 **(6)**	70	20 **(12)**	18
Demand	5	8	7	14	

2. Create closed loop for unoccupied cells

Unoccupied cell	Closed path	Net cost change
S_1D_2	$S_1D_2 \rightarrow S_1D_4 \rightarrow S_3D_4 \rightarrow S_3D_2$	30 - 10 + 20 - 8 = 32
S_1D_3	$S_1D_3 \rightarrow S_1D_4 \rightarrow S_3D_4 \rightarrow S_3D_2 \rightarrow S_2D_2 \rightarrow S_2D_3$	50 - 10 + 20 - 8 + 30 - 40 = 42
S_2D_1	$S_2D_1 \rightarrow S_2D_2 \rightarrow S_3D_2 \rightarrow S_3D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	70 - 30 + 8 - 20 + 10 - 19 = 19
S_2D_4	$S_2D_4 \rightarrow S_2D_2 \rightarrow S_3D_2 \rightarrow S_3D_4$	60 - 30 + 8 - 20 = 18
S_3D_1	$S_3D_1 \rightarrow S_3D_4 \rightarrow S_1D_4 \rightarrow S_1D_1$	40 - 20 + 10 - 19 = 11
S_3D_3	$S_3D_3 \rightarrow S_3D_2 \rightarrow S_2D_2 \rightarrow S_2D_3$	70 - 8 + 30 - 40 = 52

There is no negative net cost, thus it reaches optimal solution

The minimum total transportation cost =19×5+10×2+30×2+40×7+8×6+20×12=743

MODI

(Modified Distribution Method)

MODI

- MODI adalah versi yang lebih efisien dari Stepping Stone. Ia menggunakan **rumus matematis** untuk menghitung *opportunity cost* tanpa harus membuat loop untuk setiap sel kosong.
- Menilai optimalitas dan memperbaiki solusi dengan cara yang lebih efisien.

MODI Method Steps (Rule)	
Step-1:	Find an initial basic feasible solution using any one of the three methods NWCM, LCM or VAM.
Step-2:	Find u_i and v_j for rows and columns. To start a. assign 0 to u_i or v_j where maximum number of allocation in a row or column respectively. b. Calculate other u_i's and v_j's using $c_{ij} = u_i + v_j$, for all occupied cells.
Step-3:	For all unoccupied cells, calculate $d_{ij} = c_{ij} - \left(u_i + v_j\right)$, .
Step-4:	Check the sign of d_{ij} a. If $d_{ij} > 0$, then current basic feasible solution is optimal and stop this procedure. b. If $d_{ij} = 0$ then alternative soluion exists, with different set allocation and same transportation cost. Now stop this procedure. b. If $d_{ij} < 0$, then the given solution is not an optimal solution and further improvement in the solution is possible.
Step-5:	Select the unoccupied cell with the largest negative value of d_{ij}, and included in the next solution.

Step-6:	Draw a closed path (or loop) from the unoccupied cell (selected in the previous step). The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell.
Step-7:	1. Select the minimum value from cells marked with (-) sign of the closed path. 2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell). 3. Add this value to the other occupied cells marked with (+) sign. 4. Subtract this value to the other occupied cells marked with (-) sign.
Step-8:	Repeat Step-2 to step-7 until optimal solution is obtained. This procedure stops when all $d_{ij} \geq 0$ for unoccupied cells.

MODI

1. Hitung nilai potensi (u dan v) untuk baris dan kolom:
   - Pilih salah satu baris atau kolom dan tetapkan nilai potensinya (misalnya u1 = 0).
   - Gunakan rumus:

     $C_{ij} = u_i + v_j$

   untuk sel yang sudah dialokasikan, lalu cari nilai u dan v lainnya.
2. Hitung nilai opportunity cost (dij) untuk sel kosong:
   - Rumus:

     $d_{ij} = C_{ij} - (u_i + v_j)$
3. Evaluasi nilai dij:
   - Jika semua dij ≤ 0 → solusi optimal.
   - Jika ada dij > 0 → pilih sel dengan Δdijterbesar dan lakukan re-alokasi seperti pada metode Stepping Stone.

Iteration 1

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

1. Find initial solution (VAM / NWCR / LCM)

	D_1	D_2	D_3	D_4	Supply	Row Penalty
S_1	19(5)	30	50	10(2)	7	9 | 9 | 40 | 40 | -- | -- |
S_2	70	30	40(7)	60(2)	9	10 | 20 | 20 | 20 | 20 | 40 |
S_3	40	8(8)	70	20(10)	18	12 | 20 | 50 | -- | -- | -- |
Demand	5	8	7	14		
Column Penalty	21 21 -- -- -- --	22 -- -- -- -- --	10 10 10 10 40 40	10 10 10 50 60 --		

total transportation cost
=19×5+10×2+40×7+60×2+
8×8+20×10
=779

Iteration 1

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30	40 **(7)**	60 **(2)**	9
S_3	40	8 **(8)**	70	20 **(10)**	18
Demand	5	8	7	14	

2. Find ui and vj for all occupied cells(i,j), where cij=ui+vj

u is used to represent row, whereas v is used to represent column. Substituting, v4=0

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 **(5)**	30	50	10 **(2)**	7	$u_1 = 10$
S_2	70	30	40 **(7)**	60 **(2)**	9	$u_2 = 60$
S_3	40	8 **(8)**	70	20 **(10)**	18	$u_3 = 20$
Demand	5	8	7	14		
v_j	$v_1 = 9$	$v_2 = -12$	$v_3 = -20$	$v_4 = 0$		

3. Find dij for all unoccupied cells(i,j), where dij=cij-(ui+vj)

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 **(5)**	30 [32]	50 [60]	10 **(2)**	7	$u_1 = 10$
S_2	70 [1]	30 [-18]	40 **(7)**	60 **(2)**	9	$u_2 = 60$
S_3	40 [11]	8 **(8)**	70 [70]	20 **(10)**	18	$u_3 = 20$
Demand	5	8	7	14		
v_j	$v_1 = 9$	$v_2 = -12$	$v_3 = -20$	$v_4 = 0$		

Choose the minimum negative value from all dij (opportunity cost) = d22 = [-18]

4. Draw a closed path from S2D2

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30 [32]	50 [60]	10 **(2)**	7
S_2	70 [1]	30 [-18] **(+)**	40 **(7)**	60 **(2)** **(-)**	9
S_3	40 [11]	8 **(8)** **(-)**	70 [70]	20 **(10)** **(+)**	18
Demand	5	8	7	14	

Select minimum allocated value among all negative position (-) on closed path = 2

5. Substract 2 from all (-) and Add it to all (+)

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30 **(2)**	40 **(7)**	60	9
S_3	40	8 **(6)**	70	20 **(12)**	18
Demand	5	8	7	14	

Repeat the step 2 to 5, until an optimal solution is obtained

Iteration 2

	D_1	D_2	D_3	D_4	Supply
S_1	19 **(5)**	30	50	10 **(2)**	7
S_2	70	30 **(2)**	40 **(7)**	60	9
S_3	40	8 **(6)**	70	20 **(12)**	18
Demand	5	8	7	14	

2. Find ui and vj for all occupied cells(i,j), where cij=ui+vj

Substituting, u1=0

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 **(5)**	30	50	10 **(2)**	7	$u_1 = 0$
S_2	70	30 **(2)**	40 **(7)**	60	9	$u_2 = 32$
S_3	40	8 **(6)**	70	20 **(12)**	18	$u_3 = 10$
Demand	5	8	7	14		
v_j	$v_1 = 19$	$v_2 = -2$	$v_3 = 8$	$v_4 = 10$		

Iteration 2

3. Find dij for all unoccupied cells(i,j), where dij=cij-(ui+vj)

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 **(5)**	30 [32]	50 [42]	10 **(2)**	7	$u_1 = 0$
S_2	70 [19]	30 **(2)**	40 **(7)**	60 [18]	9	$u_2 = 32$
S_3	40 [11]	8 **(6)**	70 [52]	20 **(12)**	18	$u_3 = 10$
Demand	5	8	7	14		
v_j	$v_1 = 19$	$v_2 = -2$	$v_3 = 8$	$v_4 = 10$		

Since all dij≥0. So final optimal solution is obtained

The minimum total transportation cost =19×5+10×2+30×2+40×7+8×6+20×12=743

Latihan

- Carilah salah satu study kasus tentang Transportation Problem, Tentukan *Decision variable*:, *objective function*, dan *constraints* dari studi kasus tersebut.
- Selanjutnya selesaikan masalah Transportation Problem itu dengan minimal 2 Metode Solusi awal dan lanjutkan dengan metode Solusi optimal dengan menggunakan Stepping Stone dan MODI